TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 2 2017

Section 1 Calculator Free

Functions and Sketching Graphs

STUDENT'S NAME: SOLUTIONS

DATE: Tuesday 7th March **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (30 marks)

For the function $f(x)=\dfrac{x^2-2x+1}{2(x+1)}$

(a) Determine $f(0)$. [1]

$f(0)=\frac{1}{2}$ ✓ (y-intercept)

(b) State the domain of the function. [1]

$\mathbb{R}$, $x\neq -1$ ✓

(c) Determine the real roots (zeros) for the equation $f(x)=0$. [2]

x^2-2x+1
$=(x-1)(x-1)$
$=(x-1)^2$ ✓

$f(x)=0$ when $x=1$ ✓

This is a repeated (non-distinct) root thus the graph will 'touch' the x-axis (Tangental Turning Point)

(d) Determine the coordinates and nature (max or min) of any turning points. [4]

Set: $(2x-2)\cancel{2}(x+1)-(x-1)^2\cancel{2}=0$ ✓ ie. Numerator of Quotient Rule.

$\Rightarrow (x-1)(2(x+1)-(x-1))=0$

$\Rightarrow (x-1)(x+3)=0$

$\therefore x=1$ or $x=-3$ ✓

as expected from part (c)

$\therefore$ min. when $x=1$ i.e. at $(1,0)$ ✓

Confirm with 1st Derivative sign test

max. when $x=-3$ i.e. at $(-3,-4)$ ✓

Confirm with 1st Derivative sign test

$f(-3)=\frac{16}{-4}$
$=-4$

(e) State any asymptotes for the function. [3]

Vertical asymptote (pole) at $x = -1$. ✓

$$2x+2 \overline{)\, x^2 - 2x + 1} \quad \frac{1}{2}x - \frac{3}{2}$$

$x^2 + x$

$-3x + 1$

$-3x - 3$

4 ✓

$\therefore f(x) = \frac{1}{2}x - \frac{3}{2} + \frac{2}{x+1}$

$\therefore$ oblique asymptote:

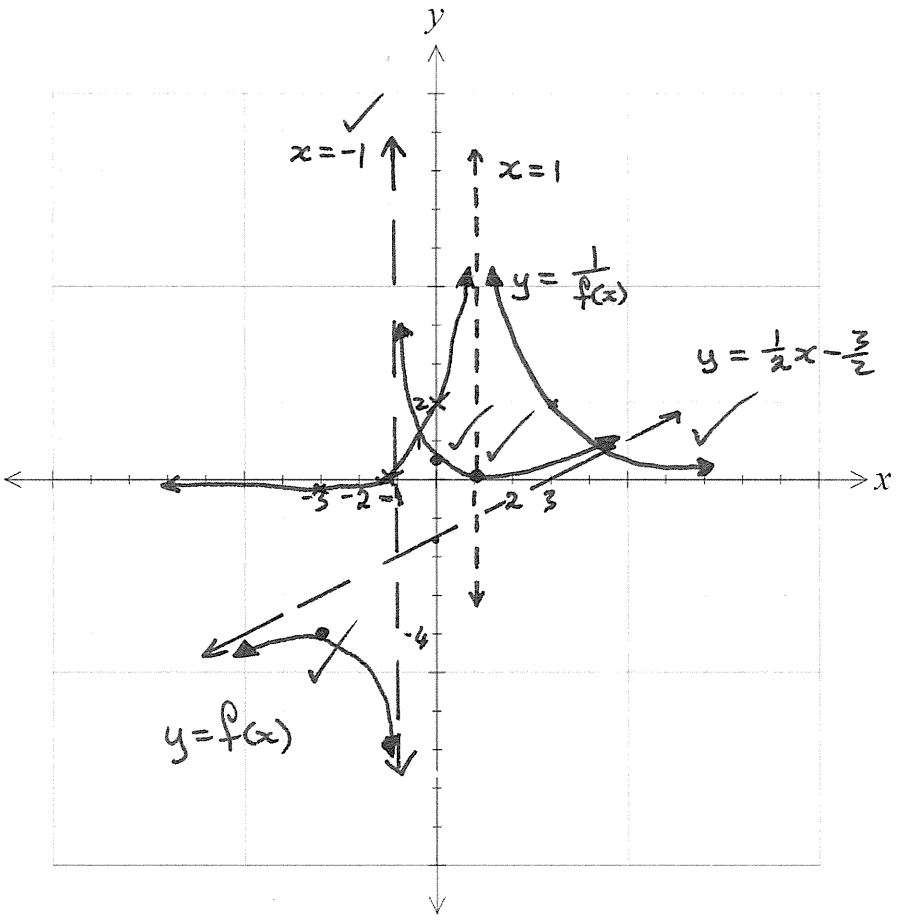
$y = \frac{1}{2}x - \frac{3}{2}$ ✓

(f) Complete the following statements: [2]

As $x \to \infty$, $f(x) \to \infty$ ✓ as $y = \frac{1}{2}x - \frac{3}{2} \to \infty$

As $x \to -1^+$, $f(x) \to \infty$ ✓ i.e. approaching pole from right hand side.

(g) Sketch the graph of the function, clearly labelling all the above features. [5]



(h) State the range of the function. [2]

Range $= \{y : y \leq -4 \text{ or } y \geq 0\}$ ✓ ✓

(i) What type of relationship is this function? [1]

Many to One (m-1) ✓

(j) Does $f^{-1}(x)$ exist? If so, why? If not, why not? [2]

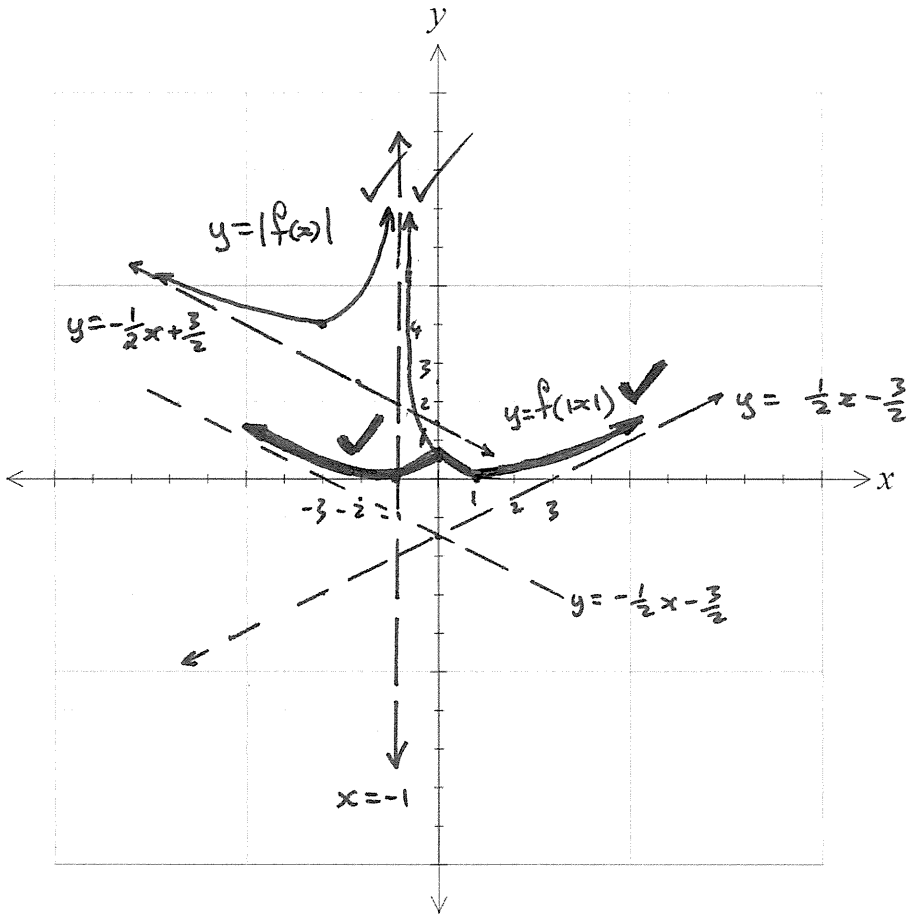
$f^{-1}(x)$ does not exist, fails the 'horizontal' line test. ✓
ie. m-1
(only 1-1 have inverses)

(k) Graph and label $y = \left(f(x)\right)^{-1}$ on the same set of axes above. [3]

$= \frac{1}{f(x)}$ ie. the reciprocal of $f(x)$.

Reciprocating: Pole when $x = 1$ and x-intercept when $x = -1$ ✓
y-intercept $\frac{1}{2} = 2$
Min. at $(-3, -\frac{1}{4})$; Horizontal asymptote $y = 0$ (x-axis)

(l) Graph and label $y = |f(x)|$ and $y = f(|x|)$ on the set of axes below. [4]



End of Questions

TRINITY COLLEGE

Mathematics Specialist Units 3 & 4
Test 2 2017

Section 2 Calculator Assumed *

Functions and Sketching Graphs

STUDENT'S NAME: SOLUTIONS

DATE: Tuesday 7th March **TIME:** 20 minutes **MARKS**: 20

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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* N.B. Use (and make sense) of your calculator (it can't think) throughout.

2. (4 marks)

If $f(x)=\dfrac{1-x}{|x-1|}$ and $g(x)=\dfrac{1}{x}$, state:

(a) The domain and range for $f(x)$. [2]

Domain: $x \neq 1$ ✓

Range: $y = \pm 1$ ✓

(b) State the necessary minimum restriction on the natural domain of $g(x)$ so that $y=f(g(x))$ exists. [2]

Natural domain of $g(x)$ is $x \neq 0$

This needs to be further restricted to $x \neq 1$ ✓✓

since $g(1)=1$ and $f(g(x))=\dfrac{1-\frac{1}{x}}{|\frac{1}{x}-1|}$

$f(g(1))$ produces $\frac{0}{0}$ indeterminate

3. (4 marks)

For the function $f(x)=\dfrac{1}{1-x}-1$, determine the inverse function $f^{-1}(x)$.

Let: $y=\dfrac{1}{1-x}-1$

Interchange x and y : ✓

$x=\dfrac{1}{1-y}-1$

Use ClassPad (CAS) to solve for y: Transpose ✓✓

$y=\dfrac{x}{x+1}$

$\therefore\ f^{-1}(x)=\dfrac{x}{x+1}$ ✓

Using algebra skills

$\Rightarrow\ x+1=\dfrac{1}{1-y}$

$\Rightarrow\ \dfrac{1}{x+1}=1-y$

$\Rightarrow\ y=1-\dfrac{1}{x+1}$

$\therefore\ f^{-1}(x)=1-\dfrac{1}{x+1}$

$=\dfrac{x}{x+1}$ ✓

4. (4 marks)

Given that $f(g(x))=x^2+4x+13$ and $f(x)=x^2+9$, determine the rule for $g(x)$.

$f(g(x))=x^2+4x+4+9$ ✓

$=(x+2)^2+9$ ✓

$\therefore\ g(x)=x+2$ ✓✓

5. (4 marks)

Given $f(x)=\dfrac{x}{x+1}$, solve for x if $3f(x)+f\left(\dfrac{1}{x}\right)=2$

$\Rightarrow \dfrac{3x}{x+1}+\dfrac{\frac{1}{x}}{\frac{1}{x}+1}=2$ ✓✓

$\therefore x=1$ solve on ClassPad.

✓✓

$\Rightarrow \dfrac{3x}{x+1}+\dfrac{1}{1+x}=2$

$\Rightarrow \dfrac{3x+1}{x+1}=2$

$\Rightarrow 3x+1=2x+2$

$\therefore x=1$ as above.

This may be quicker than typing the above into ClassPad.

6. (4 marks)

Solve the following:

(a) $|2x+1|=|x-5|$ [2]

$\therefore x=-6,\ x=\frac{4}{3}$ ✓✓

There are several algebraic approaches, but here ClassPad is quickest. (CAS)

(b) $|2x-3|\geq 2$ [2]

$\therefore x\leq\frac{1}{2}$ or $x\geq\frac{5}{2}$

✓ ✓

N.B. This question could easily be placed in Section 1. Whilst you could use algebra, a manual graphing approach would be best.

End of Questions